How Do We Mitigate Against a Marauding Terrorist?

Problem presented by

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CAST

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Executive Summary

In recent years, worldwide terrorist strategy has changed from long term planning and high impact attacks, to relatively short-term planning, 'marauding' attacks. In other words, terrorist attacks from 2010 to present have often consisted of a small number of terrorists in densely populated public spaces actively searching out people to maim and kill with weapons that are harder to regulate, such as knives and vehicles, with little regard for their own survival. This begs the question, therefore, of how we mitigate against these 'marauding terrorists'. If we assume 3 different types of actors: Public, Terrorists, and Responders, and assume the strategy of the Terrorists is to kill as many people as possible before they themselves are killed, how can the strategies of the Public and Responders (police, bouncers etc.) be optimised to minimise loss of life?

In this report, the problem at hand and important information are compiled before 3 approaches to model a terrorist attack in a public space are considered - a Particle Model, a Discrete Network Model, and a Game Simulation model. Whilst this is by no means a complete list of possible models of a terrorist attack, these were believed to be models that could be developed the most in a week at the ESGI130 at the University of Warwick. For each model type, we first consider their assumptions and suitability to the problem, then model the scenario. Finally, we consider possible extensions to each model, and also how they may be used to evaluate the most effective strategies of the Public and Responders given the information available to them at any one time.

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1 Problem Statement

1.1 Outline

This decade, 'marauding terrorist' attacks, where a small group of terrorists move around a public space with weapons such as knives, guns, and vehicles, have been on the rise. Given the small amount of planning needed to perform these attacks, except for acquiring the weapon of choice and determining the location, it is very hard to prevent such attacks, and therefore the focus has to primarily turn to mitigation. How can such a situation be mitigated against? Firstly, before this can be answered in any meaningful way, the scenario needs to be described in greater detail.

In an attack, there are three types of actors: Public, Terrorists, and Responders. Public are the actors that are being targeted and killed, and they receive information about events either by witnessing it themselves, by conversing with one another and Responders, or perhaps by digital means (e.g. social media). Furthermore, it is assumed that their behaviour follows the government guideline of "Run, Hide, Tell". Terrorists on the other hand, are the actors targeting the Public, and will receive information events by witnessing it themselves, or talking to one another. Their behaviour is to main and kill as many Public actors as possible until they are killed or apprehended themselves. Lastly, Responders are policeofficers (on and off-duty), bouncers, independent members of the public etc. who try to apprehend or kill the terrorists as fast as possible with the aim to limit the loss of life of the Public. They gather information in the same way as Public actors, as well as talking to Public actors themselves.

With these actors defined and a greater knowledge of their behaviours, then the initial question about mitigation becomes: given the number of Public, Terrorists, and Responders, as well as the weapons of the Terrorists and the location of the attack, what is the optimal strategy of a limited number of Responders that minimises total loss of life? Furthermore, if the Public strategy of "Run, Hide, Tell" is no longer assumed, what are the optimal strategies of the Public and Responders that minimises loss of life?

Before approaches are considered to model the scenario, everything that should ideally be considered by any model is synthesized and summarised.

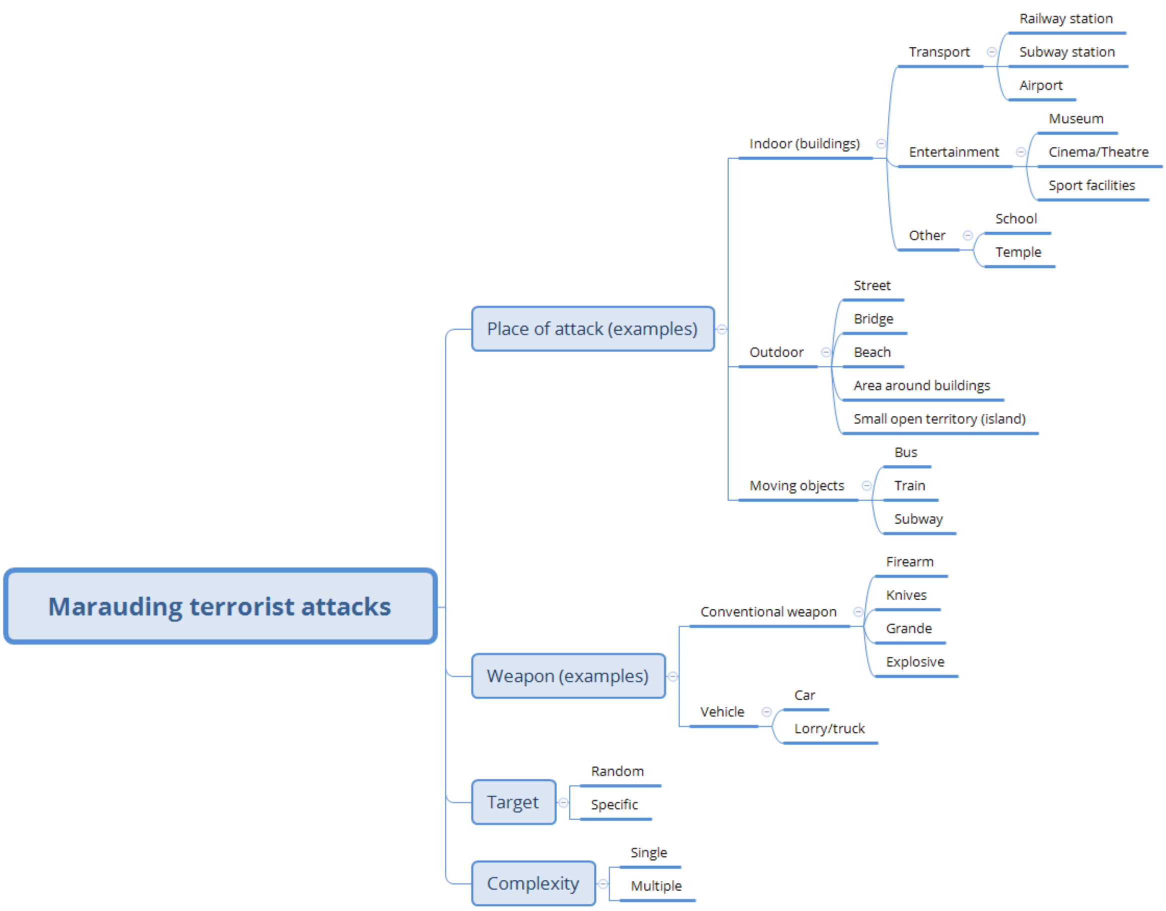
1.2 Knowledge synthesis

During the ESGI workshops, gathered knowledge was synthesized and visualized with mind maps and issue trees.

1.2.1 Marauding terrorist attack

Figure 1 considers all of the constituent parts of a terrorist attack: the location, the type of weapon, the type of target and the complexity of an attack. Furthermore, the location is characterised by its features, which may increase the likelihood of an attack there. See Figure 2.

Figure 1: Mind map of marauding terrorist attack



1.2.2 Public knowledge

Figure 3 includes different examples of public information that influences their decision making process, and their behaviour of "Run, Hide, Tell". This excludes information about responders. The public knowledge, and as well as public actors' behaviour may change after getting information about responders' status e.g. responders' location, number, and weaponry.

Except for public knowledge, it could prove important to incorporate the responders' knowledge about attack. This information is likely to be helpful when modelling the decision making of the responders.

1.2.3 Examples of data in to the model

Figure 4 includes different examples of information that can be used to:

Figure 2: Mind map of features of attack place

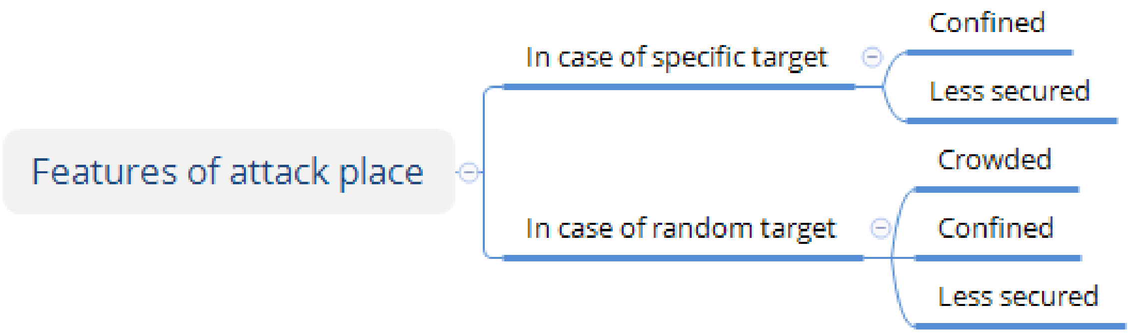
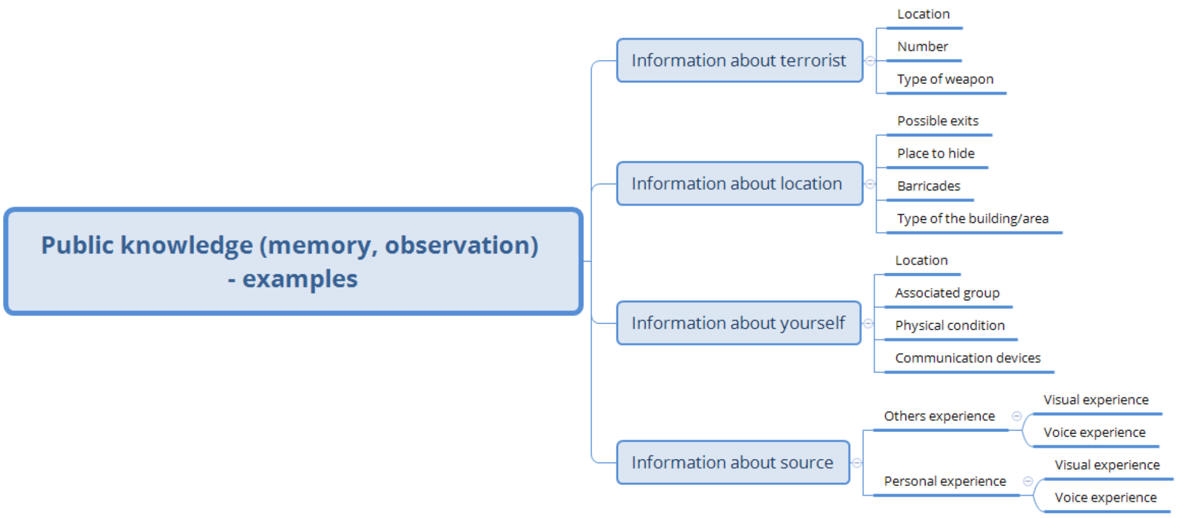
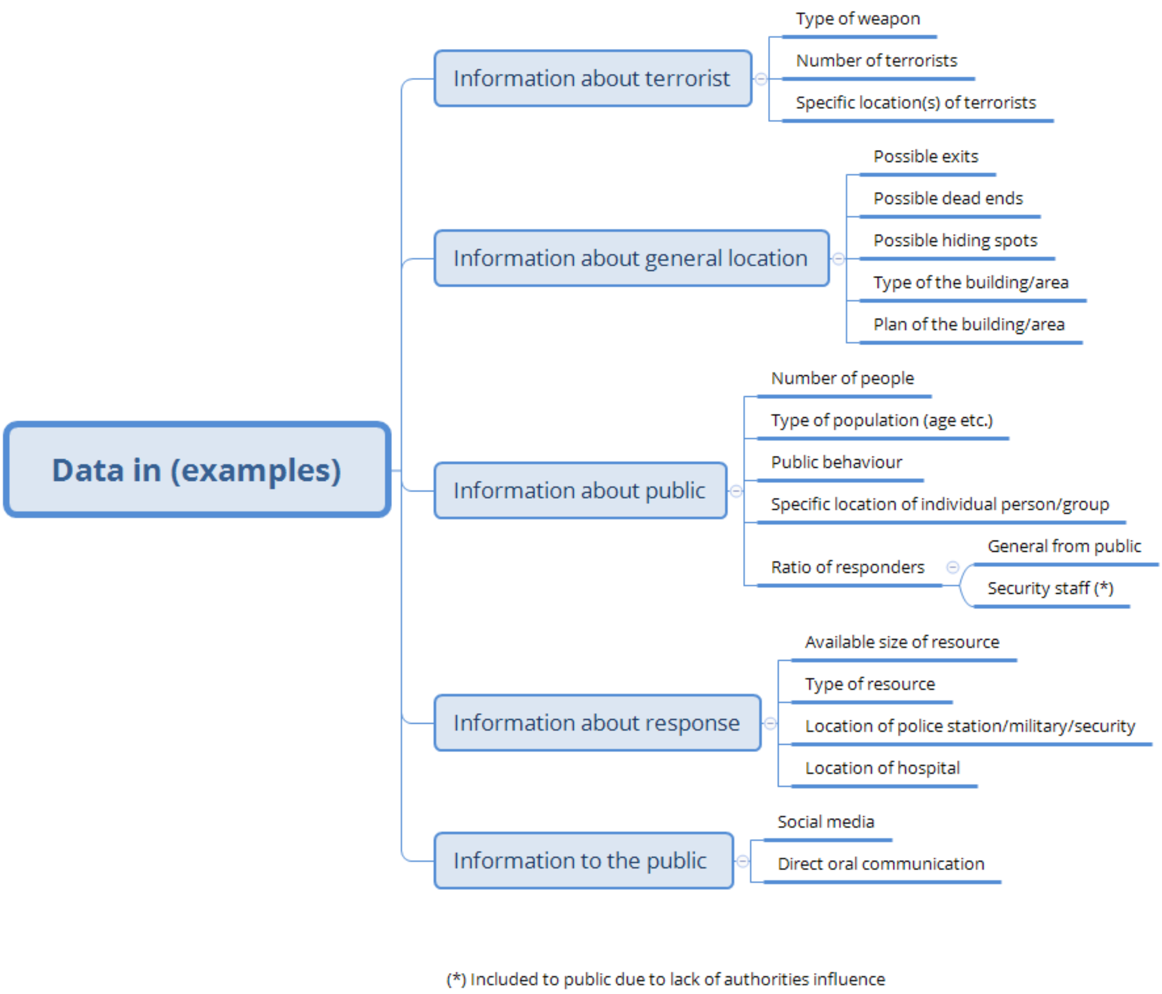


Figure 3: Mind map of public knowledge



- Create a model of a panicked crowd's movement
- Make decisions about the right response – for example the number of responders and how they behave as groups and individually
- Check the influence of sharing information between the public and responders

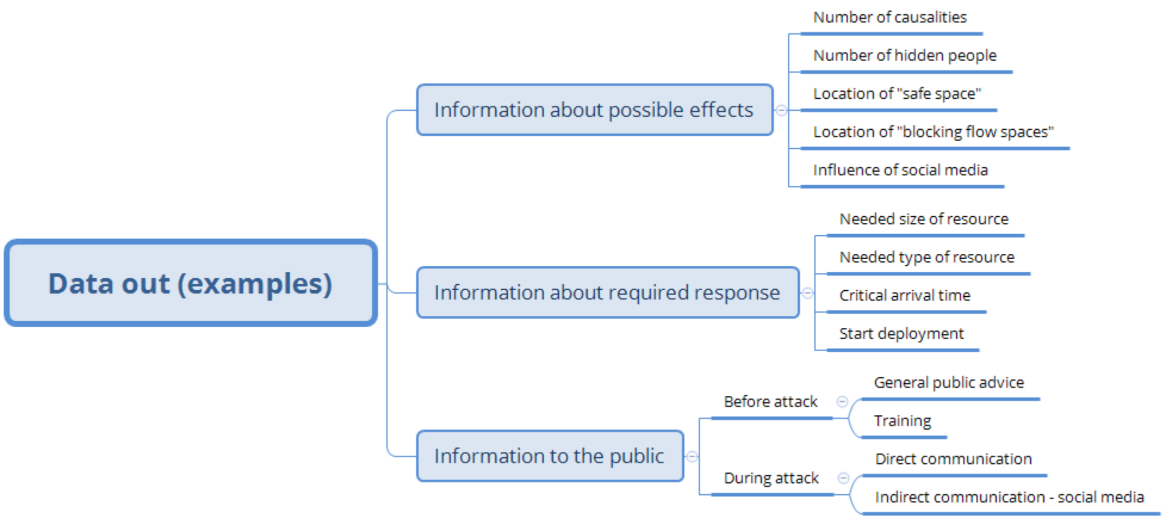
Figure 4: Mind map of “data in” - model.



1.2.4 Examples of data out of the model

Figure 5 describes what information a model may be able to provide to best inform responders and the public. The first branch considered is 'Information about possible effects' of different marauding terrorist attack types, as in Figure 1. The other branches are 'Information about required response' and 'Information to the public'. This could inform future advice to prepare responders before and during a terrorist attack. Moreover, it could lead to advice given to the public e.g. using social media.

Figure 5: Mind map of "data out" - model.



2 Solution Approaches

2.1 Particle model

Presented in this chapter is a particle model to analyse the movement of civilians and marauding terrorists in a two-dimensional domain such as a bridge, room, or city centre. The model is based on the already broad amount of literature regarding crowd and herd movement [1, 4, 7], including elements of panic [2, 5, 6]. The model consists of each public actor α being modelled as separate particles with a position $\mathbf{x}_\alpha$, a velocity $\mathbf{v}_\alpha$, and an acceleration $\mathbf{a}_\alpha$. The positions and velocities of the public actors then evolve due to the acceleration of the public actors which is governed using Newton's second law of motion, $F = ma$, where F is the force on the public actor, m is the mass, and a is the acceleration of the public actor. The system of ODEs that governs the position and velocity is thus

$$\frac{\mathrm{d}\mathbf{x}_\alpha}{\mathrm{d}t} = \mathbf{v}_\alpha, \tag{1}$$

$$\frac{\mathrm{d}\mathbf{v}_\alpha}{\mathrm{d}t} = \frac{1}{m_\alpha}\mathbf{f}_\alpha, \tag{2}$$

where m_α is the mass of public actor α (which herein is taken to be one for simplicity) and $\mathbf{f}_\alpha$ is the force acting on public actor α. Lastly, as the public actors are physical objects with a finite size, they are modelled as spheres with a radius r_α.

The forces acting on each public actor are dependent on their surroundings and their motives. The forces that are included in existing models in the literature depend heavily on the situation that is being modelled. One force that is almost always necessary is the repulsion of public actors from physical obstacles such as walls and pillars. There is also usually a desired direction the public actors wish to go and therefore a force draws them towards this point. In large crowds, another force that is often included is that individuals do not wish to be too close to other people, due to social limits. Lastly, there is a tendency for individuals in crowds to tend to align their velocities with others that are nearby, in a similar fashion to animals in herds. From this point, the rest of the forces that one could include depend on the psychological model being used for public actors in the particle model; the possible inclusions that could be added to these forces are discussed in section 2.1.4.

For this example, forces describing the desire to be away from the terrorist, denoted as $\mathbf{f}_\alpha^{\text{terrorist}}$ and towards any exits that are nearby, denoted by $\mathbf{f}_\alpha^{\text{attract}}$ are included. Also included are the repellent nature of physical obstacles and other public actors denoted as $\mathbf{f}_\alpha^{\text{obstacle}}$ and $\mathbf{f}_\alpha^{\text{repel}}$, respectively, and the herd-like mentality denoted as $\mathbf{f}_\alpha^{\text{align}}$. Lastly, humans are not governed by these forces exactly and so there is some random noise associated to the forces, denoted by $\xi(t)$. Therefore, the force on public actor α, denoted by $\mathbf{f}_\alpha$, is expressed as a sum of all the forces and motives discussed, giving us

$$\mathbf{f}_\alpha = \mathbf{f}_\alpha^{\text{terrorist}} + \mathbf{f}_\alpha^{\text{attract}} + \mathbf{f}_\alpha^{\text{obstacle}} + \mathbf{f}_\alpha^{\text{repel}} + \mathbf{f}_\alpha^{\text{align}} + \xi(t). \tag{3}$$

2.1.1 Forces acting on public actors

The exact mathematical forms of all these forces are largely debatable, however, as they are generally inversely related to the distance – something close to a public actor will tend to affect their movement more than something at a great distance. The forms of $\mathbf{f}_\alpha^{\text{obstacle}}$ and $\mathbf{f}_\alpha^{\text{repel}}$ used here are similar to those used in [4]. The obstacle force is felt by public actor α relative to the closest obstacle point, $\mathbf{x}_w$, if the distance to that point, $d_w = -\|\mathbf{x}_w - \mathbf{x}_\alpha\|$, is smaller than a chosen threshold distance. The force is described as

$$\mathbf{f}_\alpha^{\text{obstacle}} = -\left[f_{\max}^{\text{obstacle}} \frac{1}{1 + (d_w/r_\alpha)^{p_1}} + g_{\max}^{\text{obstacle}} \exp\left(\frac{-d_w}{\sigma_{\text{obstacle}}} \right) \right] \frac{\mathbf{x}_w - \mathbf{x}_\alpha}{d_w}, \tag{4}$$

as the force felt by the public actor α from the closest point of the obstacle $\mathbf{x}_w$. The first term in the brackets is taken from [4], which is mid-range and this represents the desire of people to not be too close to walls, and the second term was added to have a very large, very localised, force adjacent to the wall to represent a public actor's inability to move through walls. The maximum constants $f_{\max}^{\text{obstacle}}$ and $g_{\max}^{\text{obstacle}}$ are the maximum values the mid-range and short-range forces can take, respectively. The variables that govern how quickly these forces decay as the public actor is further away from the obstacle are p_1 for the mid-range force and σ_{obstacle} for the short-range force. The coefficients are such that $g_{\max}^{\text{obstacle}} \gg f_{\max}^{\text{obstacle}}$ and in this case $p_1 = 2$ in order to have relatively fast decay from the obstacle, and $\sigma_{\text{obstacle}} = 0.1$ in order that the force be localised at the obstacle.

The repellent force is split into a near-range force, for when a public actor comes within a short distance from public actor α, and a contact force for when a public actor comes within r_α of another public actor [7]. The force is described as

$$\mathbf{f}_\alpha^{\text{repel}} = -\sum_{\beta \neq \alpha} \left(f_{\max}^{\text{repel}} \frac{1}{1 + \rho_\alpha^{p_2}} \mathbf{s} + a g_{\max}^{\text{repel}} \frac{1}{1 + \rho_{\alpha\beta}^{p_3}} \mathbf{s} \right), \tag{5}$$

where $\mathbf{s}$, ρ_α, $\rho_{\alpha\beta}$, and a are given by

$$\mathbf{s} = \frac{\mathbf{x}_\beta - \mathbf{x}_\alpha}{\|\mathbf{x}_\beta - \mathbf{x}_\alpha\|}, \tag{6}$$

$$\rho_\alpha = \frac{\|\mathbf{x}_\beta - \mathbf{x}_\alpha\|}{r_\alpha}, \tag{7}$$

$$\rho_{\alpha\beta} = \frac{\|\mathbf{x}_\beta - \mathbf{x}_\alpha\|}{r_\alpha r_\beta}, \tag{8}$$

$$a = 1 \quad \text{if} \quad \rho_{\alpha\beta} \leq 1 \quad \text{and} \quad a = 2 \quad \text{if} \quad \rho_{\alpha\beta} > 1. \tag{9}$$

The near-range force in the left-hand side term in (5) while the contact force is the right-hand side term.

In the case of a single terrorist and single exit, forces repelling public actors from the terrorist and attracting the public actors to the exit are included in a similar way to the obstacle force (with a long-range and a short-range force). Let $\mathbf{x}_T$ and $\mathbf{x}_{\text{exit}}$ denote the locations of the terrorist and exit, respectively. Then $d_T = \|\mathbf{x}_T - \mathbf{x}_\alpha\|$ denotes the distance of public actor α from the terrorist and $d_{\text{exit}} = \|\mathbf{x}_{\text{exit}} - \mathbf{x}_\alpha\|$, the distance of public actor α from the exit. Then, the terrorist and exit forces are given by

$$\mathbf{f}_\alpha^{\text{terrorist}} = -\left[f_{\max}^{\text{terrorist}} \exp\left(\frac{-d_T}{\sigma_{\text{terrorist}}^{\text{long}}} \right) + g_{\max}^{\text{terrorist}} \exp\left(\frac{-d_T}{\sigma_{\text{terrorist}}^{\text{short}}} \right) \right] \frac{\mathbf{x}_T - \mathbf{x}_\alpha}{d_T}, \tag{10}$$

$$\mathbf{f}_\alpha^{\text{attract}} = \left[f_{\max}^{\text{attract}} \exp\left(\frac{-d_{\text{exit}}}{\sigma_{\text{attract}}^{\text{long}}} \right) + g_{\max}^{\text{attract}} \exp\left(\frac{-d_{\text{exit}}}{\sigma_{\text{attract}}^{\text{short}}} \right) \right] \frac{\mathbf{x}_{\text{exit}} - \mathbf{x}_\alpha}{d_{\text{exit}}}, \tag{11}$$

where for both forces $\sigma^{\text{short}} \ll \sigma^{\text{long}}$. Here, σ^{short} and σ^{long} help determine the decay rate of f and g.

Lastly, a flocking force [5] is used to model the force that causes the public actors to prefer to move at a velocity similar to the velocities of the public actors nearby. The force influencing public actor α to align their velocity with the public actors around them is

$$\mathbf{f}_\alpha^{\text{align}} = \sum_{\beta \neq \alpha} \frac{f_{\max}^{\text{align}}}{1 + \|\mathbf{x}_\alpha - \mathbf{x}_\beta\|^2} \frac{\mathbf{v}_\beta}{\|\mathbf{v}_\beta\|}, \tag{12}$$

where $f_{\max}^{\text{align}}$ is the aligning force parameter, which is usually smaller than other forces.

Of course, the public actors have only a finite speed and acceleration. Therefore, the speed of every public actor is limited to $v_{\max}$ (around 8 ms^{-1}) but this could easily be specific to each public actor instead. Thus, the normalised velocity of public actor α is given by the following smoothed minimum function:

$$\mathbf{v}_\alpha := -\log\left[\exp(-v_{\max}) + \exp(-\|\mathbf{v}_\alpha\|)\right] \frac{\mathbf{v}_\alpha}{\|\mathbf{v}_\alpha\|}. \tag{13}$$

The acceleration is also limited in a similar way, although people slow down much faster than they speed up and so the acceleration is limited according to the dot product of the acceleration with the velocity to distinguish between acceleration and deceleration.

Let $a_{\max}$ be the maximum acceleration magnitude if both acceleration $\mathbf{a}_\alpha$ and velocity are in the same direction (around 5 ms^{-2}). Let the maximum deceleration magnitude be $k \times a_{\max}$, i.e. if acceleration and velocity are in opposite direction. For all other directions, the maximal acceleration magnitude $A_{\max}$ is given by

$$A_{\max} = a_{\max} \left(\frac{k+1}{2} + \frac{k-1}{2} \frac{\mathbf{a}_\alpha \cdot \mathbf{v}_\alpha}{\|\mathbf{a}_\alpha\| \|\mathbf{v}_\alpha\|} \right). \tag{14}$$

As is done for the velocity, the acceleration is normalised using a smoothed minimum function such as (13).

Heuristics are used to describe the strategies used by public actors when trying to avoid multiple terrorists or choose between multiple exits. In the case of multiple terrorists, closer terrorists must be considered as a higher threat. In the case of n_T terrorists, the total terrorist force is defined as a weighted sum of the individual forces

$$\mathbf{f}_{\alpha,\text{total}}^{\text{terrorist}} = \frac{1}{\sum_{i=1}^{n_T} w_i} \sum_{i=1}^{n_T} w_i \mathbf{f}_{\alpha,Ti}^{\text{terrorist}}, \tag{15}$$

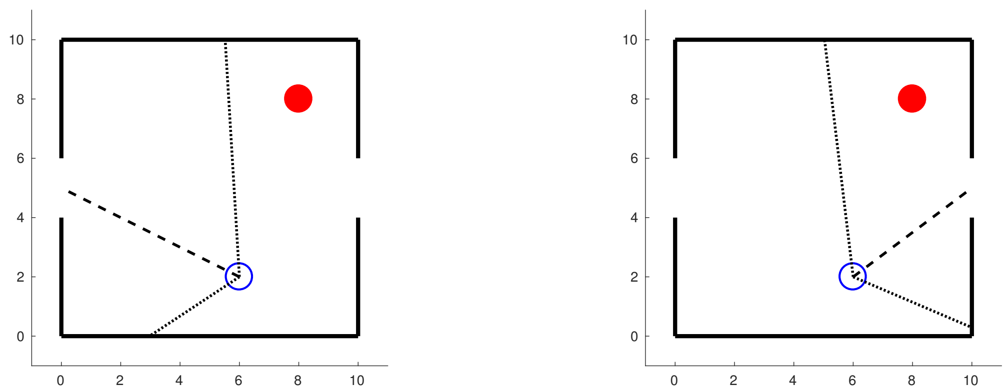
with weights

$$w_i = \frac{1}{\|\mathbf{x}_\alpha - \mathbf{x}_{Ti}\|^2}, \tag{16}$$

where $\mathbf{x}_{Ti}$ denotes the position of terrorist i and $\mathbf{f}_{\alpha,Ti}^{\text{terrorist}}$, their repulsive force as defined in (10).

In the case of multiple exits, public actors should be attracted towards the closest exit that is not blocked by a terrorist. In the case of a single terrorist, public actor α considers an exit to be blocked if the terrorist stands within a cone between the exit and the public actor. If no exit is deemed safe, the public actor is attracted to the closest exit. In Figure 6, the right-hand side exit is not considered safe since it is within the cone. Indeed, the public actor will choose the left-hand side exit even if it is further.

Figure 6: Illustration of the heuristic used to determine if an exit is safe. The left-hand side exit is considered safe because the terrorist (red) is outside the cone between the public actor (blue) and the exit. Similarly, the right-hand side exit is not considered safe.



The angle of the cone is chosen to be 120°. Hence, an exit is considered safe if

$$\frac{(\mathbf{x}_{\text{exit}} - \mathbf{x}_\alpha) \cdot (\mathbf{x}_T - \mathbf{x}_\alpha)}{\|\mathbf{x}_{\text{exit}} - \mathbf{x}_\alpha)\| \|\mathbf{x}_T - \mathbf{x}_\alpha\|} > 0.5 = \cos(60°). \tag{17}$$

In the case of multiple terrorists, using this heuristic for each terrorist often causes all exits to be deemed unsafe. Instead of comparing the positions of the public actor and exits with respect to individual terrorist, they are compared to a relative centre of the terrorists. Instead of using the individual positions $\mathbf{x}_{Ti}$ in (17), a weighted centre of mass of the form is used

$$\mathbf{x}_T = \frac{1}{\sum_{i=1}^{n_T} w_i} \sum_{i=1}^{n_T} w_i \mathbf{x}_{Ti}, \tag{18}$$

where w_i are defined by (16).

2.1.2 Terrorist Movement

The position $\mathbf{x}_T$ and velocity $\mathbf{v}_T$ are modelled using a similar particle model as that described above. The position $\mathbf{x}_T$ is governed by

$$\frac{\mathrm{d}\mathbf{x}_T}{\mathrm{d}t} = \mathbf{v}_T, \tag{19}$$

$$\frac{\mathrm{d}\mathbf{v}_T}{\mathrm{d}t} = \frac{1}{m_T}\mathbf{f}_T, \tag{20}$$

where the mass of a terrorist is also set to $m_T = 1$. The forces acting on the terrorist are similar to those for the public actors but with a different desired direction of travel and without the aligning force as the terrorist does not want to move with the crowd. The force on the terrorist $\mathbf{f}_T$ is given by

$$\mathbf{f}_T = \mathbf{f}_T^{\text{obstacle}} + \mathbf{f}_T^{\text{repel}} + \mathbf{f}_T^{\text{closest}}. \tag{21}$$

The force from the obstacles $\mathbf{f}_T^{\text{obstacle}}$ is exactly the same as for the public actors as the obstacles interact exactly the same with terrorists as they do with public actors. The terrorists do not want to be close to each other, therefore the short-range repellent force $\mathbf{f}_T^{\text{repel}}$ influences the terrorists in the same way as the public actors.

The desired direction of the terrorists, described by $\mathbf{f}_T^{\text{attract}}$, is more difficult to model since one would need to know a priori the objective of the terrorists and the weapons used in the attack. Several terrorist strategies (and therefore directions of travel) were considered, but it is assumed here that the terrorists move towards the closest public actor with a magnitude proportional to how far away they are. Let $\mathbf{x}_{\text{closest}}$ be the position of the closest public actor to position $\mathbf{x}_T$, and $d_c = \|\mathbf{x}_{\text{closest}} - \mathbf{x}_T\|$. Thus,

$$\mathbf{f}_T^{\text{closest}} = f_{\max}^{\text{terrorist}} \exp\left(\frac{-d_c}{\sigma_{\text{closest}}}\right) \frac{\mathbf{x}_T - \mathbf{x}_{\text{closest}}}{d_c}, \tag{22}$$

where σ_{closest} is large enough that the force decays very slowly.

The terrorist also needs to have finite speed and acceleration so these are limited, again using (13) and (14). However, it is reasonable to think that the terrorists are more fit than the average public actor. Therefore, it is assumed that a terrorist has a relatively higher maximum speed and therefore $v_{\max,\mathrm{T}} > v_{\max}$.

Assuming that if the terrorist comes within some distance d_{kill}, the public actor dies, becomes inactive and no forces are produced by the dead public actor. With a small d_{kill}, this models a knife or vehicle attack. In the case of firearms, the attractive force would most likely have to consider more than simply the closest public actor, and a large lethality radius around terrorists is likely to be an over-simplification.

2.1.3 Results

The model described in section 2.1 is implemented on MATLAB [8] using Euler's method [3] for time stepping the position and velocity of the public actors and the terrorist.

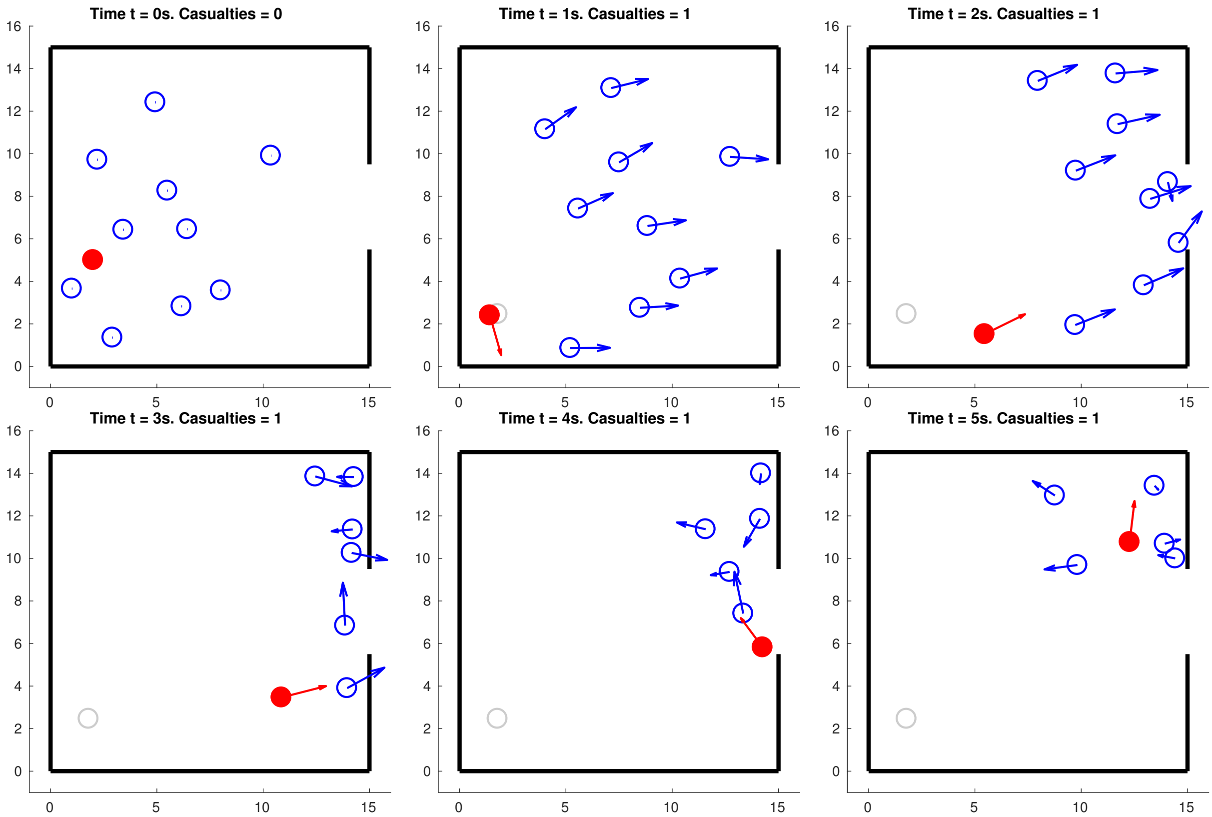
The topology used in the first experiment is a small square room with one exit and one terrorist on the other side of the room with the starting positions of the ten public actors randomly placed using a uniform distribution across the room. In Figure 7, the movement of the different agents is illustrated over 5 seconds. One can observe how public actors first head for the exit but move away from it when the terrorist's repulsive force becomes stronger than the attractive force of the exit. One can also see in the first 2 seconds, the public actors in the upper part of the room have a positive y-velocity. Since the terrorist started close to them, they initially run away from the terrorist instead of directly heading towards the exit. This causes them to be trapped in the corner.

The second experiment takes place in a larger room with two exits, one terrorist in the middle, and fifty public actors with uniformly distributed initial positions. In Figure 8, one can observe how the public actors split in two groups heading for different exits. When a terrorist blocks the left-hand side exit in the third second, the public actors start heading for the other exit. Again, in the last frames, one can see a few public actors returning to the left-hand side exit once a terrorist starts blocking the right-hand side exit.

The third experiment take place in a square room with a single exit, three terrorists in the middle, and fifty public actors with uniformly distributed initial positions. In Figure 9, one can observe how public actors do not head for the single exit when terrorists are blocking it.

The last experiment takes place in a long domain representing a bridge for which either side is considered an exit. As in previous experiments, there are three terrorists in the middle and fifty public actors with uniformly distributed initial positions. In Figure 10, one can observe how much easier it is for the public actors to escape when exits are so wide. Indeed, there is absolutely no clogging at the exits.

Figure 7: Simulation in a 15m × 15m room with 1 exit, 10 public actors and 1 terrorist.



2.1.4 Future work

From these simulation results, one can see that these particle and crowd models illustrate one way that people could move in a crowded room and when trying to escape a moving threat. The results of the simulations can be very different depending on which parameters are used in the forces - for example, each $f_{\max}$, $g_{\max}$, σ, p, ρ, and the random noise ζ. This means that the model needs to be somewhat calibrated to fit the heuristics people have about crowd movement from experience (i.e. model parameter settings should be informed by behavioural experts). Additionally, it was observed that the parameters are somewhat dependent on the geometry of the situation, which is undesirable. An early adjustment to the model would be to find other expressions for the forces which make the results independent from the geometry, or understand how they depend on the geometry and include this into the expressions.

Despite the calibrating of the parameters in the model being both tedious and relying heavily on heuristic knowledge, it can be helpful as one can use the changes in the parameters to predict what happens when the behaviour of the public actors changes. Using the average number of casualties from several simulations using these parameters, changing the actors' starting positions in each simulation, one could potentially advise public actors about what to do in these situations to minimise public actor casualties. For example, if by varying $f_{\max}^{\text{terrorist}}$ and $f_{\max}^{\text{attract}}$ and measuring the average number of casualties from several simulations, one can advise the public actors how much to prioritise heading

Figure 8: Simulation in a 30m × 20m room with 2 exits, 50 public actors and 1 terrorist.

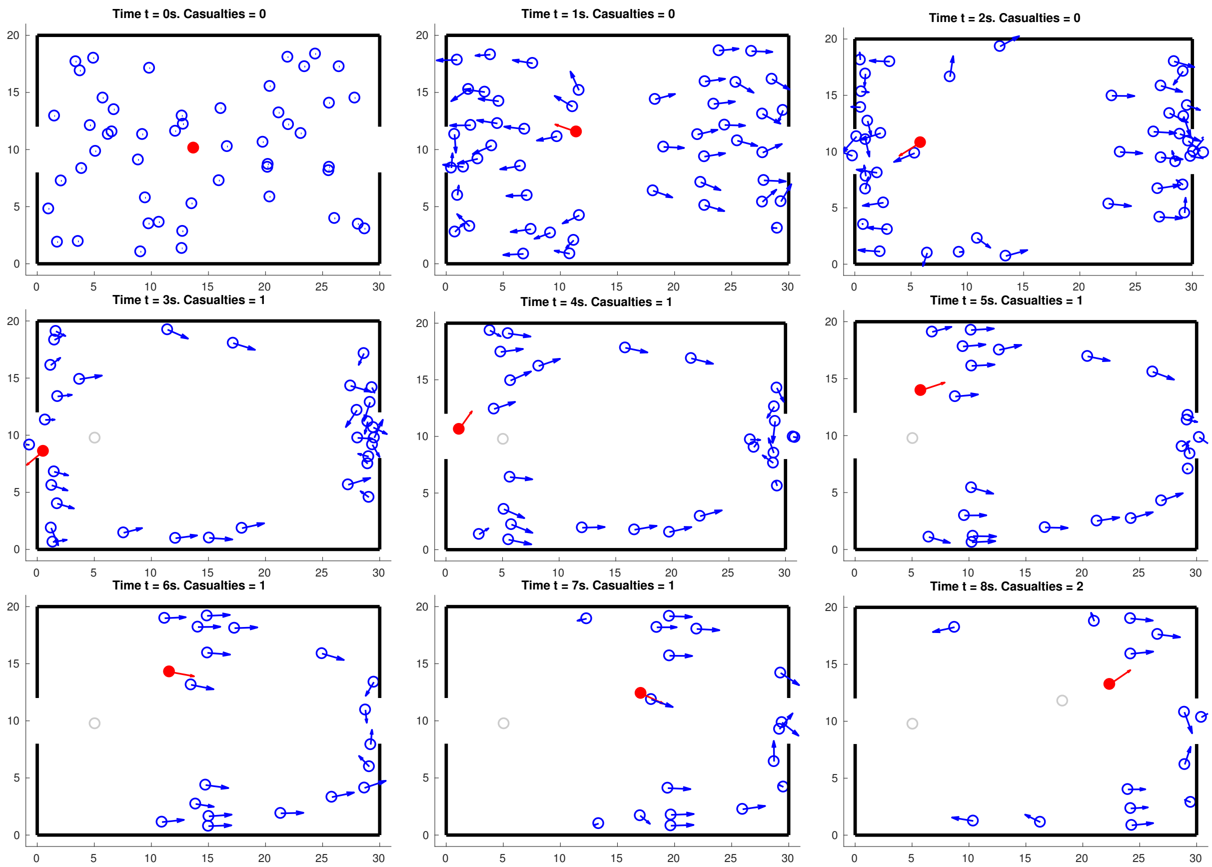


Figure 9: Simulation in a 20m × 20m room with 1 exit, 50 public actors and 3 terrorists.

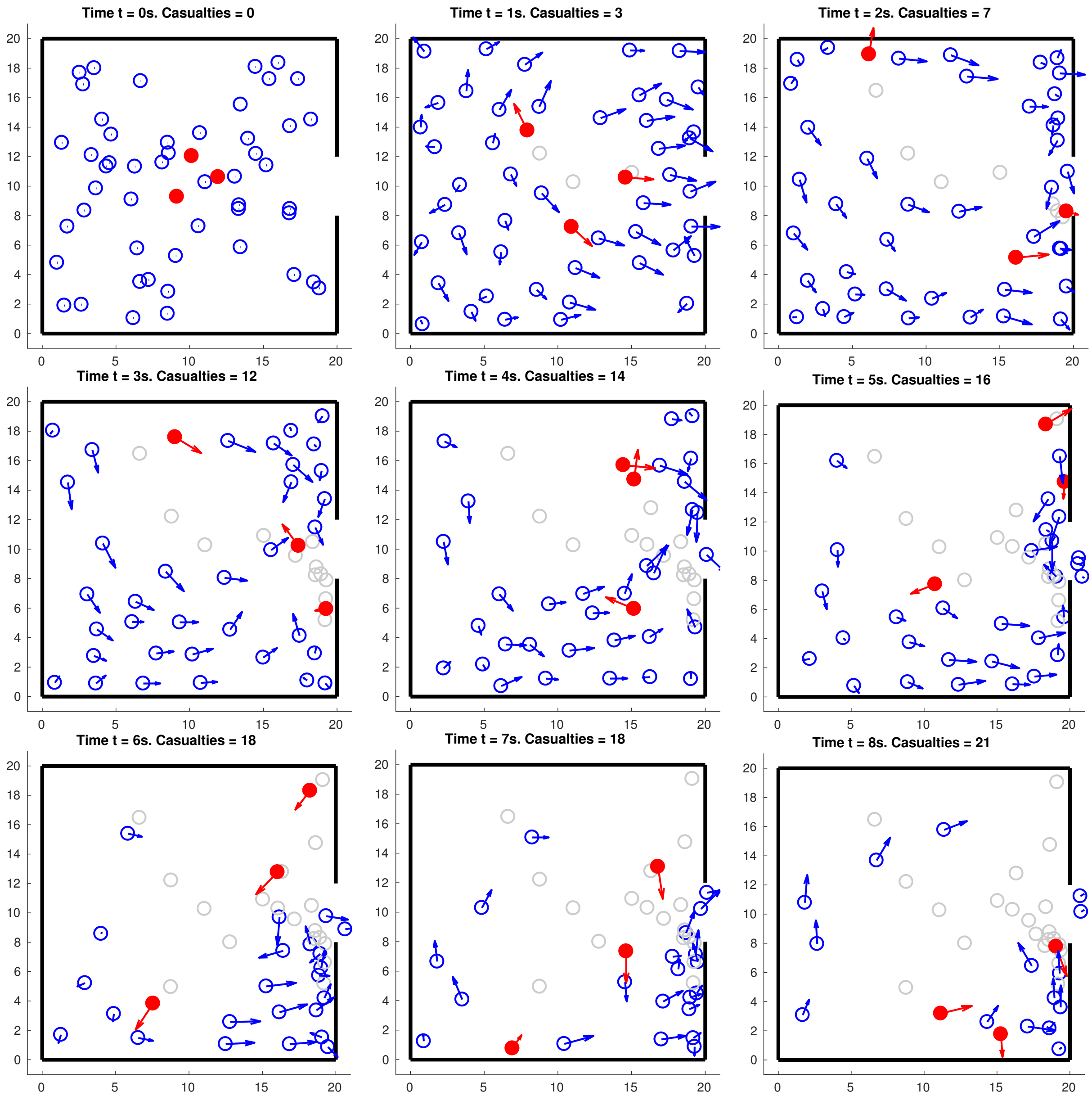
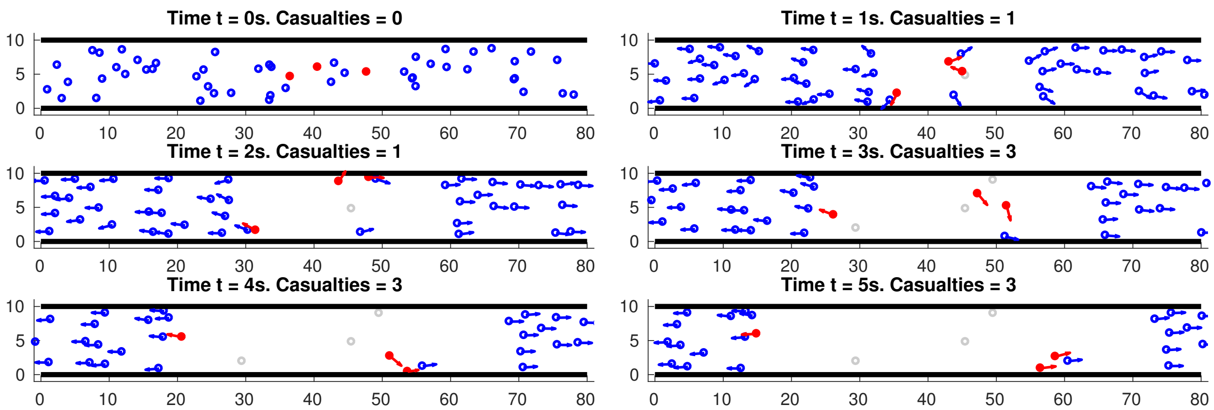


Figure 10: Simulation on a 80m × 10m bridge with 50 public actors and 3 terrorists.



for an exit over immediately getting as far away from the terrorist as possible.

While the current model does include important phenomena found in crowd models [1, 4, 7] and herd models [5], there are many other features that could be included. This is largely due to the fact that these preexisting models are for much simpler situations than that being considered here. Indeed, crowd models do not account for strong non-local repulsive forces and prey-predator models do not include obstacles and exits. The strategies of the different actors in a marauding terrorist model are much more complex. The discussions about what to include and how are not necessarily a mathematical question but more down to psychological modelling. Further behaviours could be included mathematically into the model using similar force terms as presented in section 2.1 depending on what is driving the force and where from. Examples of further considerations to the model include how a public actor makes a decision when there are multiple threats, if there is a chance the public actor is unhurt by the terrorist, if they attempt to attack the terrorist and if they choose to hide. As discussed above, this last inclusion could then be used to vary the parameters in the model and quantify how useful the "Run, Hide, Tell" advice currently given to public actors caught in a terrorist attack.

In conclusion, the work carried out on the particle model was to demonstrate how this could be achieved, and to highlight the huge amount of literature available on the subject of crowd movement and panic modelling. The particle-based model presented here can be adapted and extended to be as sophisticated as is deemed necessary and can be applied to many situations and geometries of locations. This could be used as a way of calibrating the model in a more rigorous fashion by using data and evidence from previous attacks to ensure the model is realistic in a terrorist attack.

2.2 Discrete time network model

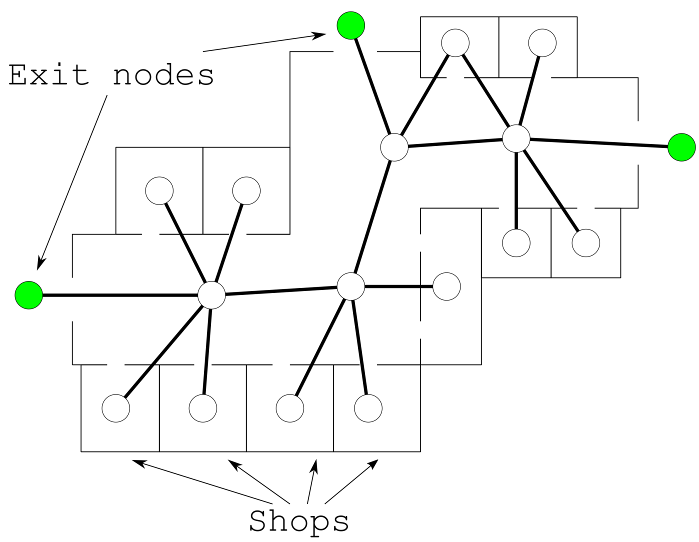
2.2.1 Principles and algorithm outline

The progress of a marauding terrorist attack is modeled in discrete time steps (which for brevity we shall call *turns*)

The spatial domain represented as a graph. The vertices of the graph represent physical locations, and two locations share an edge if *actors* (civilians, terrorists, responders, etc.) can travel between them in one turn. See Figure 11.

A key feature of this model, which distinguishes it from existing work on panicking crowd and predator-prey models that we found in an initial literature survey, is that individuals (and populations) behave according to information that spreads between populations in a node and also remotely (via phone calls to authorities, responder radio communications, social media). Actors keep a record (either individually or as populations) of locations (nodes) and times when other actors have been seen or heard, and may exchange this information via various mechanisms with allies. The behaviour of an individual or population is determined by the information available to them, their environment, and a pre-defined strategy.

Figure 11: Representing a shopping centre as a network.



Note that to simplify the implementation non-integer numbers of public actors (i.e. civilians) is allowed in the code provided. This simplification is referred to as the *continuum model*. To enforce only integer populations, a probabilistic approach may be required to determine the motion of the actors. An example of such a probabilistic model is given in section 2.3.

A turn involves the following key steps, only some of which have been implemented in this model:

1. All actors evaluate neighbouring nodes, based on the information available to them, the intrinsic values of the locations, and a strategy. Using these evaluations and specified decision rules, each able actor may move to a neighbouring node or stay at the node they currently occupy. Note in the continuum case, replace 'each' with 'a proportion of".

2. In addition to movements, during a turn we may observe the following actions:
   - Terrorists inflict casualties to those within a node, or possibly in neighbouring nodes, depending on the scale and weapon.
   - Unaware civilians become alerted, for example if they share a node with other alerted civilians or responders, or by receiving social media messages.
   - Alerted civilians go into hiding if there is space in the node.
   - Hiding civilians send information to the authorities or to social media.
   - Hiding civilians become "runners" again, depending on nearby threats and responders.
   - Alerted or hiding civilians become responders.
   - Responders apprehend/damage terrorists in the same or neighbouring nodes.

3. Terrorists generate sightings that alert nearby civilians and responders, who then add to their information.
4. Responders generate sightings that nearby attackers and civilians add to their information.
5. Civilians generate sightings that nearby terrorists and reponders add to their information.

2.2.2 Remark on space/time scales

The model described here is based on the assumption that nodes of the graph represent locations that can be traversed by an actor in one time step. If this leads to an unrealistic model of the location, or infeasible computation, we suggest the following:

- To represent smaller spaces (finer resolution) use a smaller time step, or allow actors to traverse more than one edge per step. Either will most likely be computationally expensive to simulate a given time in the real world
- To represent larger spaces (lower resolution) take a larger time step, or do not allow actors to move every turn. With the simplification of modeling civilians as a continuum or probability distribution, this can also be achieved by having a greater fraction staying at its current node. For example, if p_t^s is the fraction of the (surviving) population that occupy node N at time s and 'intend' to move to neighbouring node M at time t, motion at 1/2 a node per turn could be modeled by taking
$$\begin{aligned} q_t &= p_t^{t-2} + p_t^{t-1}/2 \\ p_{t+1}^{t-1} &= p_t^{t-1}/2 - d_t, \end{aligned} \tag{23}$$
where q_t is the population that makes this move, and d_t depends on casualties taken at N at time t, for example
- An alternative method to allow for differing rates of travel would be to introduce extra edges and nodes within our graph. The proportion of actors making a step in the new graph which previously corresponded to two steps in the old graph would be very small. This would represent a particularly fast actor. Similarly, the proportion of actors that move less than what corresponds to one step in the old graph would be very small. This would represent a particularly slow actor.

2.2.3 Mathematical description: a simple "danger" function

As previously described, the geometry of the location is represented by a directed graph with the sets of vertices, and edges denoted by V and E respectively.

Before giving a detailed mathematical framework it is useful to consider the following example, that demonstrates how a population of public actors at a certain node might behave. The purpose of this example is to provide intuition for the formal system put forward afterwards. It should not be interpreted as a final model.

Fix a vertex v_i, to all vertices v_j adjacent to v_i, the following quantity is associated:

$$\text{Danger at } v_j = \sum_{n=0}^{5} \frac{\text{number of terrorists observed at } v_j \, n \text{ turns ago}}{n}. \tag{24}$$

Note that a greater number of terrorists at node v_j increases the perceived danger, and that older observations are given less weight, in fact those older than 5 turns are ignored.

Let v_{j^*} be the vertex adjacent to v_i for which this number is minimised. In other words, v_{j^*} has the least danger among all neighbours of v_i, this turn.

Then the system could evolve via the simple rule: "all alert public actors at v_i move to v_{j^*}".

2.2.4 Mathematical description: general model

The mathematical model is now put forward in a rigorous manner. Any edge $e \in E$ is assigned a length L_e. For a vertex v, α_v, β_v denote the population capacity and number of available hiding spaces in v, respectively. Other properties could also be added, such as modifiers for damage or visibility for actors in the node.

Different edge sets could be defined for different groups of actors. For example some edges may be available for actors on foot, that cannot be traversed by actors using a vehicle.

The number of turns elapsed (time) is denoted by t. For each t, denote the population in vertex $v_i \in V$ by

$$P_i^t = (U_i^t, A_i^t, H_i^t, R_i^t, T_i^t)^\top.$$

The components $U_i^t, A_i^t, H_i^t, R_i^t, T_i^t$ represent the numbers of un-alert public, alert public ("runners"), hiding public, responders and terrorists, respectively.

For each of these sub-populations $A_i^t, \dots, T_i^t$ except un-alerted public, there is a corresponding *information set*, $I_{A,i,t}, \dots, I_{T,i,t}$, consisting of a set of ("memories"), that those actors hold.

Each of these memories has the form

$$(t_0, i, n_u, n_a, n_h, n_r, n_t) \tag{25}$$

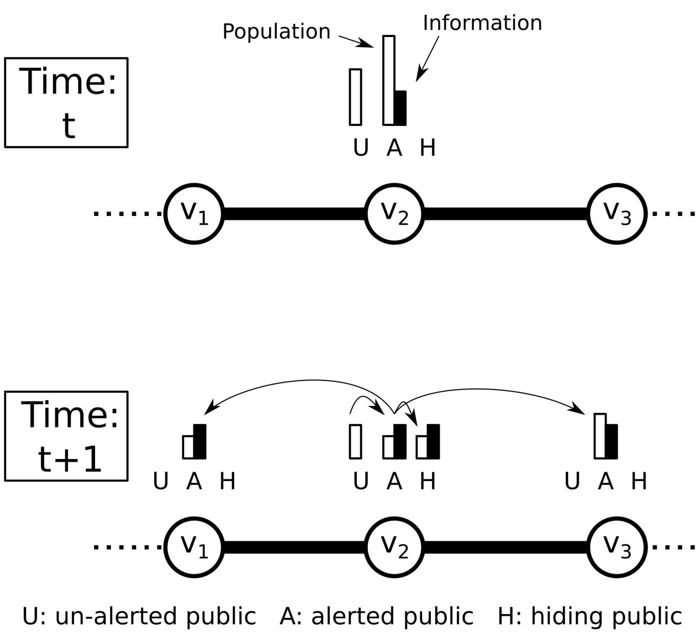
where t_0 is the observation time, i is the observed vertex, and n_u, n_a, n_h, n_r, n_t denote the populations at the place and time corresponding to the populations in P_i^t (as "observed" – which may be different from P_i^t).

The system evolution is then described as follows, for each i:

1. Calculate the population in v_i remaining there for the next turn:

$$Q_i^t = \left(P_i^t - \sum_{(i,j)\in E} X_{ij}^t P_i^t \right), \tag{26}$$

Figure 12: Movement of public actors and their information



where the (diagonal) exchange matrices

$$X_{ij}^t = X_{ij}^t(\alpha_{v_i}, \beta_{v_i}, \{P_k^t\}_{(i,k)\in E}, P_i^t, I_{A,i,t}, \ldots, I_{T,i,t}) \tag{27}$$

represent the fractions of populations moving from node v_i to node v_j at time t.

2. Re-distribute a proportion of roles amongst those remaining (e.g. unaware→run→hide, etc.), and add actors moving into v_i to the appropriate population:

$$\tilde{P}_i^{t+1} = M_i^t Q_i^t + \sum_{(j,i)\in E} X_{ji}^t P_j^t, \tag{28}$$

where $M_i^t = M_i^t(\alpha, \beta, \{P_j^{t+1}\}_{(j,i)\in E}, I_{A,i,t}, \ldots, I_{T,i,t})$ is a matrix, with each row summing to 1, that represents the re-distribution of roles (unaware, hiding, running, etc.) among the populations remaining in the node. See figure 12.

3. Calculate casualties, and apprehended terrorists etc. Subtract these losses from the corresponding populations:

$$P_i^{t+1} = \tilde{P}_i^{t+1} - D_i^{t+1}. \tag{29}$$

The casualty rate is $D_i^t = (\alpha_{v_i}, \beta_{v_i}, \{\tilde{P}_k^{t+1}\}_{(k,i)\in E}, \tilde{P}_i^{t+1})$.

4. Transport information with the movement of population:

$$\tilde{I}_{Y,i,t} = G_Y\left(\{(I_{Y,j,t}, X_{ji}^t P_j^t,) \colon (j,i) \in E\}, M_i^t, Q_i^t, I_{A,i,t}, \ldots, I_{T,i,t}\right), \tag{30}$$

where $Y = A, H, R, T$. The function G_Y should be designed to model the transmission of information between actors in the same vertex.

5. Generate new information for non-terrorists from observations and non-local means (i.e. social media, news, responder radio etc.). Merge these with current information:

$$I_{Y,i,t+1} = F_Y^i(\tilde{I}_{A,i,t}, \tilde{I}_{H,i,t}, \tilde{I}_{R,i,t}, \{P_j^{t+1}\}_{(j,i)\in E}) \cup N_Y^{t+1} \quad \text{for } Y = A, H, R. \tag{31}$$

   where N_Y^{t+1}T represents the new information given to actors in Y. The function F_Y^i should be designed to model actors observing events and populations at v_i and nearby vertices.

6. Information held by the terrorists is created and merged in the same way, but information is not shared between this group and the non-terrorists:

$$I_{T,i,t+1} = F_T^i(\tilde{I}_{T,i,t}, \{P_j^{t+1}\}_{(j,i)\in E}) \cup N_T^{t+1}. \tag{32}$$

   where N_T^{t+1} represents the new information held by the terrorists.

The details of the model are then implemented by choosing appropriate functions M_i^t, X_{ij}^t, D_i^t, G_Y, F_Y^i, and N_Y^t for $(Y = A, H, R, T)$.

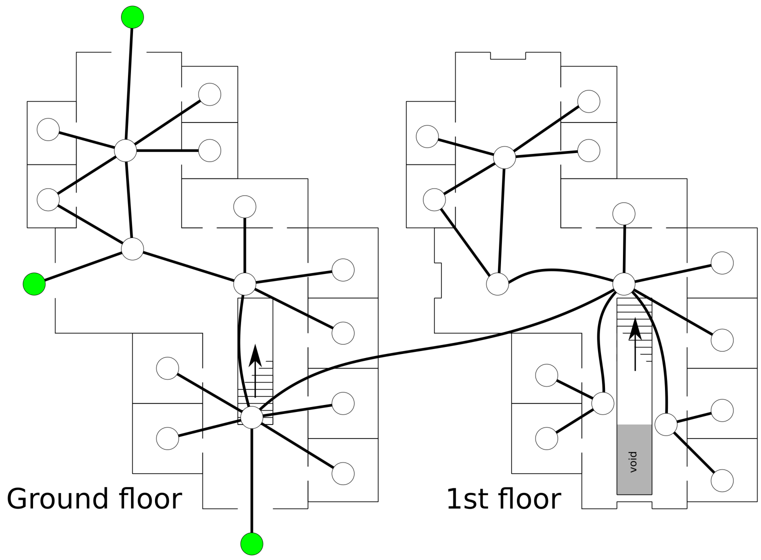
In the sample code provided, certain simple choices for these functions have been made. Note that for reasons of computational efficiency and design consideration the script does not explicitly define these functions.

2.2.5 Possible features to add

(2.1) The following features are not currently implemented in the sample code, but are potentially useful additions:

- Add different tactics and weapons for terrorists and responders. In particular there should be specific damage models and actor logic in the cases of knife, gun and vehicle attacks.
- Limit accessability of certain nodes to certain groups e.g attackers in cars.
- Assign intrinsic values to nodes for each type of actors
- Model responders sharing information via comms/radio
- Model information going to the public (social media) non-locally
- Model police radio to businesses/public information announcements or displays, as non-local means to alert/instruct the public. Similarly television and radio announcements could be modeled
- Add a model of the decision making/ information aggregation by the authorities. For example prioritising responses, analysing the effect of delays etc.
- Add the chance for civilians become responders. (May be unsuited to the continuum model.)

Figure 13: Representing multiple floors is straightforward using the network approach



- The current implementation uses a simplified method for calculating the distance between nodes. This only takes account of the physical distance between locations, rather than the length of possible routes between them, which depend on the lengths of the edges. Without the correct distance metric, actors will make unrealistic moves.
- Using a more appropriate metric (see previous remark) would allow for a wider variety of floor plans to be modeled. For example, it would be useful to be able to model buildings with multiple floors. See Figure 13.
- Allow some nodes to be designated 'safe' (exit nodes). See Figure 11.
- Introduce new states for actors, e.g. captured, injured.
- The model described above allows only for observations to nodes that are connected by an edge, i.e. locations that can be moved between within one turn. To allow for a more realistic model of observations, each node could be allocated a list of nodes that it can observe.

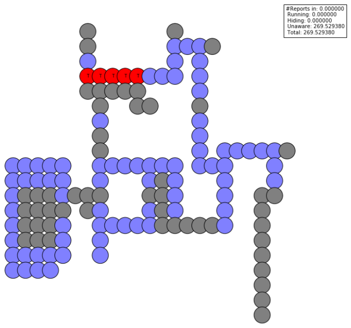
2.2.6 Potential outcomes

This model allows the simulation of an unfolding attack by a marauding terrorist. It predicts the number of people that are alive at a certain time, and the action they are currently undertaking. Importantly, modifying the functions M_i^t, X_{ij}^t, D_i^t, G_Y, F_Y^i, and N_Y^t for $(Y = A, H, R, T)$, and running multiple simulations allows some conclusions to be drawn in response to the following questions:

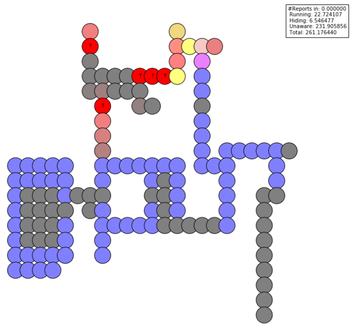
- Which strategies for the general public are more effective, i.e. run, hide/tell, fight, etc.?

Figure 14: Output of sample code. Legend: Unalert public, Alert public, Hiding public, Terrorist(s), Responder(s)

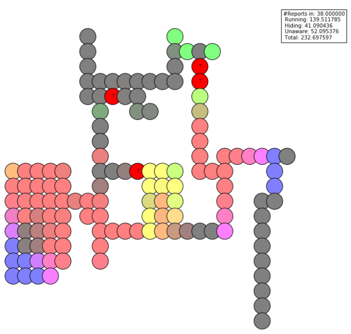
(a) $t = 0$: Attack begins



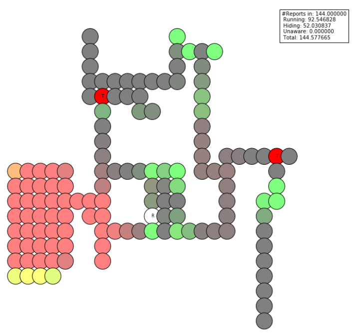
(b) $t = 3$: Public begin to run



(c) $t = 11$: Alarm spreads, some hide and call the police



(d) $t = 38$: A responder has arrived and apprehended two terrorists



- What influence will social media and public broadcasts have on how an event unfolds?
- Which types of layout make a location particularly vulnerable?
- What is an optimal response by the police? That is, how many responders should they send in?

Figure 14 comprises several images generated by an initial implementation of the model.

2.3 Game simulation model

In this section, individual strategies are monitored for each of the three groups of actors (Public, Terrorist and Responders) as states of their situation. By formulating the probability of changing state for each group, the expected number of people in each state can be generated at each point in time. This approach describes the state of person as a stochastic process, and uses discrete-time Markov chains [9]. At every point in time, an actor's state will change with given probabilities.

2.3.1 Public

A definition of a public actor's situation as stochastic process XP with states is as follows: XP = UA, R, H, T, S, DP

UA – Unaware (a person that does not know about the attack) – once they have left this state, they cannot revert to it at any point

R – Run (this could be divided into R1, R2, R3, R4 – four directions to run)

H – Hide (means just staying in the same safe for now place without information exchange)

T – Tell (situation when the person is able to pass the information about the situation to other people, responders etc.)

S – Safe (state when the person is completely safe, away from the threat) - upon entering this state, they do not change

DP – Dead (the person has been killed) - as above, upon entering this state, they do not change

Figure 15: The transition matrix – matrix of probabilities of changing state from i to j

Public	UA	R_1	R_2	R_3	R_4	H	T	S	D_P
UA									
R_1									
R_2									
R_3				p^P_{ij}					
R_4									
H									
T									
S									
D_P									

Where the probabilities $[p^p_{ij}]$ of Public actors changing from state i to state j are conditional on:

- Type of location where attack is happening (open territory or a building)
- Weapon(s) of the terrorist
- Number and location of public around
- Number and location of the terrorist(s)
- Distance from the exit/safe place
- Number and location of the responders

The probability of each state depends only on the last state the person was in, which mathematically is: $P(X_{p,n} = x_{p,n}|X_{p,n-1} = x_{p,n-1})$. hence the chain has the Markov property [9].

2.3.2 Terrorist

A definition of the terrorist's situation as stochastic process XT with states is as follows: XT = K, F, E, D

K – Kill (the terrorist stays in the place, where they can reach and kill their victims)

F – Follow (the terrorist is following people and trying to find victims. This could be divided into F1, F2, F3, F4 – four directions to follow the victims)

E – Escape (the terrorist is looking for the way out without killing people. This could be divided into E1, E2, E3, E4 – four directions to move)

DT Dead (the terrorist has been killed) - upon entering this state, the terrorist does not change

Where the probabilities $[p^t_{ij}]$ of the terrorist changing from state i to state j are conditional on:

- Type of location where the attack is happening (open territory or a building)
- Weapon(s) of the terrorist
- Number and location of public around
- Number and location of the responders
- Weapon(s) of the responders
- Escape route/location

The probability of each state depends only on the last state the person was in, which mathematically is: $P(X_{t,n} = x_{t,n}|X_{t,n-1} = x_{t,n-1})$. Hence this chain also has the Markov property[9].

Figure 16: The transition matrix – matrix of probabilities of changing state from i to j

Terrorist	K	F_1	F_2	F_3	F_4	E_1	E_2	E_3	E_4	D_T
K										
F_1										
F_2										
F_3				p^t_{ij}						
F_4										
E_1										
E_2										
E_3										
E_4										
D_T										

2.3.3 Responders

A definition of the responders' situation as stochastic process XP with states is as follows: XR = P, C, G, A, DR

P Protect (responders stays in the place to protect public there)

C Chasing the terrorist (responders are looking for the terrorist, it could be divided into C1, C2, C3, C4 – four directions to chase)

G Go out (responders are going out of the attack scene with some people from public – rescuing them, it could be divided into G1, G2, G3, G4 – four directions to move)

A Assassinate (responders assassinate the terrorist)

DR Dead (the responder has been killed) - upon entering this state, they do not change

Where the probabilities $[p^r_{ij}]$ of the responders changing from state i to state j are conditional on: type of location where attack is happening (open territory or a building), weapon of the terrorist,

- Type of location where attack is happening (open territory or a building),
- Weapon(s) of the terrorist(s)
- Weapon(s) of the responders
- Number and location of public around
- Number and location of other responders
- Number and location of the terrorist(s)
- Escape route/location.

Figure 17: The transition matrix – matrix of probabilities of changing state from i to j

Responders	K	F_1	F_2	F_3	F_4	E_1	E_2	E_3	E_4	A	D_R
K											
F_1											
F_2											
F_3				p^{r}_{ij}							
F_4											
E_1											
E_2											
E_3											
E_4											
A											
D_R											

The probability of each state depends only on the last state the person was in, which mathematically is: $P(X_{r,n} = x_{r,n}|X_{r,n-1} = x_{r,n-1})$. Hence, as expected, this chain has Markov property as well [9].

2.3.4 Estimation of probabilities

The probabilities for each actor cannot easily to be calculated directly, however they could be estimated based on: some heuristics (e.g. herd mentality), data from previous terrorist attacks, surveys (or other types of crowd-sourcing methods), and simulations.

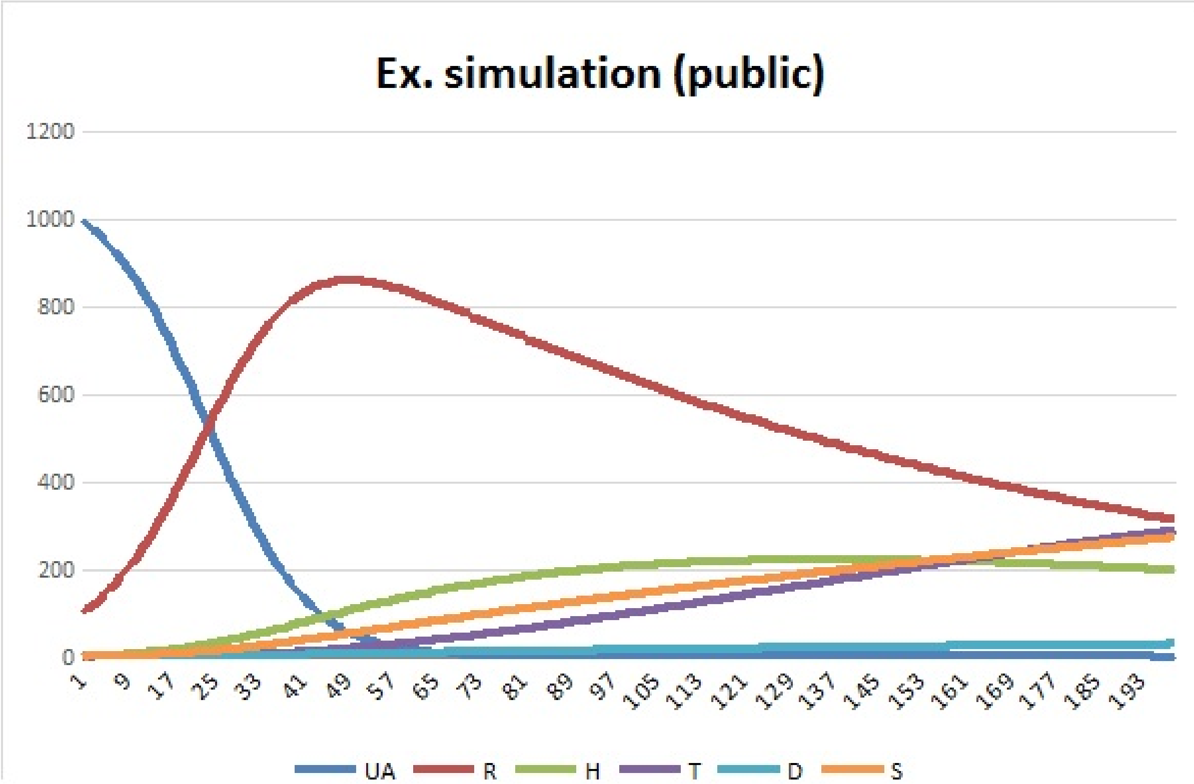
2.3.5 Possible outcome from the model

Having the initial probabilities and the initial number of actors in each state would enable a model to compute all actors' behaviour over time.

The most important goal is to minimize (or at least estimate) the number of public being killed during the terrorist attack. With the initial number of public actors in the place of attack and probabilities $[p^{p}_{ij}]$, it is possible to compute the expected number of killed public actors at each time step. Let t_n describe the time steps, where $n = 0, 1, 2, 3, \ldots$ and $E\#D_P](t_n)$ - expected number of killed public actors at t_n. Let $E\#i(t_n)$ describe the number of public actors in state i at time t_n and $[p^{p}_{ij}]$ is the probability of changing the state from i to j.

$$E\#D_P(t_0) = 0 \text{ for } n = 0, 1, 2, 3, \ldots \text{ and } E\#D_P(t_n) = \sum_{i=1} E\#i_P(t_{n-1})p^{PiD} + E\#D_P(t_{n-1}) \tag{33}$$

Figure 18: Simulation



Furthermore, the probabilities of the public and responders could be connected in some way to maximize the number of Safe and minimize the number of Dead. Armed with that information, the recommended strategy for responders (maybe also for the public) could be understood.

3 Conclusion

Whilst the report above demonstrates three not-yet-complete models of a 'maruading terrorist', these approaches clearly mark paths of research and areas for further development that could prove helpful indeed when considering what could constitute an 'optimal strategy' by Responders and Public alike in such a situation. Even if only one model is taken forward, extending it could help researchers and policy-makers better understand the dynamics of scenarios as they unfold, and how minor tweaks in strategy and tactics can have a substantial impact on the rest of the scenario, and the number of killed members of the public. Whatever happens next, due to recent societal and political observations across the world, it is likely that there will be more 'marauding terrorist' attacks to come, and the hope is that studies like this one will continue to help in whatever way they can to ultimately preserve human life.

A Appendices

A.1 Network Model Code Readme

****
Implementation of discrete-time
network model for CAST problem

Produced at ESGI 130 by Ben Pooley

Version 0.1

****

##**Script:** Script (.py) files are compatible with Python 3.5.2
##**Required Libraries:** scipy, networkx and matplotlib (All included in Anaconda 4.2.0)

##**File descriptions:**
NetworkMain1.py Sample implementation. Generate a sequence of image files illustrating the model (as implemented in Networks1.py) in the case of a single attacker , with map and initial civilian population specified in Grid2 .grd

NetworkMain2.py In this example we use the classes in Networks1.py to run a simulation on a rectangular grid, with a modified model parameters.

Networks1.py Implements primary classes to instantiate the model (both the style and functionality need significant refinement –– this should be considered proof–of–concept) N.B . Several important parameters for the behaviour of the model are specified throughout this file. In particular see " useful static parameters" (prefixed "s") at the start of class definitions

```
    Key classes specified:
        Group_base      Tracks a civilian population and
            its knowledge
        Node_base       Represents a node in the
            geometry. Responsible for managing civilian
            groups, responders, and attackers at that
            location Key methods: Update1,
            UpdateTerrorist1, UpdateResponders1
```

Terrorist Defines attacker behaviour (mostly supplementary to Node_base.UpdateTerrorist1)
Responder Defines responder behaviour (mostly supplementary to Node_base.UpdateResponders1)
Network Top-level class. Responsible for: initialising the geometry (nodes) e.g. by parsing a .grd file, calling Node_base update methods for each time step, rendering to a pyplot plot via networkx, distributing information about attacker sightings between nodes, and (currently) maintaining the authorities' knowledge base (which affects responder behaviour)

ParseBMP.py Utility script. Converts a collection of bitmaps (of the same dimensions) to a .grd file. Currently there is no interface – in/out file paths are global constants in ParseBMP.py.

Grid2.grd A plain-text file format defining network geometry (node locations and connections) as well as properties of nodes such as: Initial population, maximum node populations, and number of hiding spaces in each node.

Map.bmp Input to ParseBMP.py. Defines grid-based network layout. Non-zero RGB values correspond to nodes. Adjacent (non-diagonal) nodes will be connected (except on last row/column, due to quick coding!). N.B. A grid based layout (as produced by ParseBMP.py) is not assumed in the model, nor in the *.grd specification. For example, it may be appropriate to represent a corridor as a single node, with many connected room nodes, whereas with the ParseBMP approach, each node has degree at most 4.

Map_CAP.bmp Input to ParseBMP.py. Assumed same dimensions as Map.bmp. RGB intensity (at pixels corresponding to nodes in Map.bmp) represents civilian capacity of the node.

Map_HID.bmp Input to ParseBMP.py. Assumed same dimensions as Map.bmp. RGB intensity (at pixels corresponding to nodes in Map.bmp) represents number of hiding places in the node.

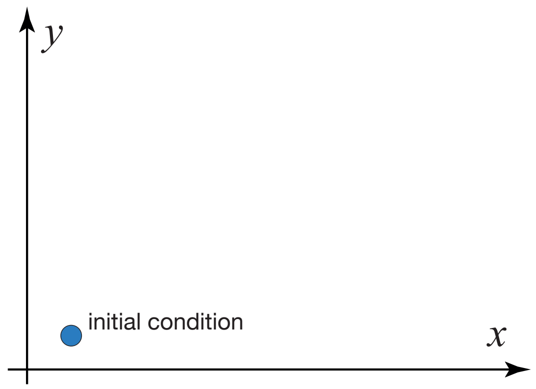
Map_POP.bmp Input to ParseBMP.py. Assumed same dimensions as Map.bmp. RGB intensity (at pixels corresponding to nodes in Map.bmp) represents initial population of (un-alerted) civilians in the node.

##Instructions to run sample NetworkMain1.py or NetworkMain2.py
Open and run the file in Spyder (supplied with Anaconda 4.2.0), or run from the command line with "python NetworkMain1.py" or "python NetworkMain2.py" (assuming PATH variables configured correctly, if using Windows)

Note: this should produce a number of image files in the working directory

##Instructions to use ParseBMP.py
Ensure that the four bitmaps Map*.bmp are in the working directory (backup Grid2.grd if required) and run ParseBMP.py as described above (using Spyder or "python ParseBMP.py" on the command line).

Figure 19: The phase space for the dynamical system, with the variables x and y, and an initial condition indicated as a blue dot.



A.2 A Continuous Dynamical Systems Model

Below is a simple relation illustrated between some fundamental elements of a generic incident. The dynamical system is a simple example of the dynamics of two competing species in its simplest nonlinear form.

We consider 2 time-dependent variables, x and y, which are both real-valued and positive. The value of x expresses both the number of terrorists and the quality of the situation as seen from the viewpoint of the terrorists, and the value of y measures the the number of victims and the quality of the situation as seen from the viewpoint of the victims.

In such a competitive species evolution, there is no stable equilibrium; the final outcome, $t \to +\infty$, sometimes called the ω-limit, will contain a finite value for only one of the dependent variables; the other variable is zero.

As initial condition $(x(0), y(0))$ in an terrorist-victim scenario, both x and y have small, positive values. This is illustrated as the blue dot in figure 19.

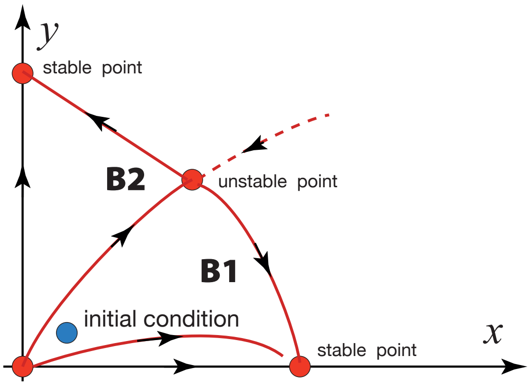
The model of the evolution of the variables has the form

$$\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= ax(t) - bx(t)^2 - cx(t)y(t) \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= dy(t) - ey(t)^2 - fy(t)x(t) \end{aligned} \tag{34}$$

The six (positive, real) parameters a, b, c, d, e, and f have the following interpretations:

- The parameter a is an indicator of the initial enthusiasm of the aggressors, as it causes the quality of the situation from their viewpoint, to improve.
- The parameter b is a measure of the strength of the ability of the responders to neutralise the terrorists. It is proportional to x^2 as it is expected to grow strongly with the number of terrorists.

Figure 20: The evolution of the initial condition in phase space depends on the location of the basins of attraction for the dynamical system. If the location of the unstable equilibria and the separatrices are such that the initial condition falls within the lower basin of attraction **B1**, the terrorists 'win' in the sense that the end state contains a finite value of x and 0 of y.

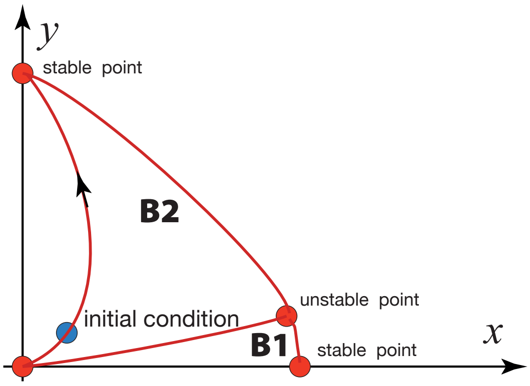


- The parameter c measures the negative impact on x of the presence of y, this is the ability of the victims (as opposed to the police responders) to neutralize the terrorists in confrontations. It will be zero if the victims only flee and hide.
- The parameter d is an indicator of the initial mobility (the ability to flee) of the victims.
- The parameter e is a measure of how bad the 'clogging' or 'jamming' effect is, in the situation at hand. Tight spaces, walls, enclosures or other hindrances to free flight, will add to the magnitude of e, and the term is proportional to y^2.
- The parameter f measures the negative impact on y of the presence of x, in other words, this is how effective the aggressors are in encounters. It is proportional to the frequency xy of encounters between species x and y.
- The structure of the phase phase flow depends on the location of the stationary points and the interlacing seperatrices that partition basins of attraction. See figure 20.

If the values of the parameters $a - f$ are such that the initial condition lies in the basin of attraction to the stable equilibrium point with a nonzero x-value, it is only a question of time before the value of y is zero, and the terrorists have achieved their goal. This is illustrated in figure 20.

On the other hand, if the parameters $a - f$, in particular the parameter b, which measures the effectiveness of the response, and to a minor extent the parameter c, which measures negative impact on the terrorist variable x of encounters with y, have values to shape the phase space flow such that the initial point is in the basin **B2**, then the evolution ends up with a finite number for the y-variable and 0 for the x-variable.

Figure 21: The phase space for the dynamical system, with the same initial situation, but now the parameters b and c favor the victims. The values force the unstable stationary point and the associated separatrices to make the initial point part of the basin **B2** of attraction to the stable equilibrium containing a finite value for y and a 0 value for x.



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